Graphing Quadratics

Any function whose equation is in the format $f(x) = ax^2 + bx + c$ (when $a \neq 0$) is called a **quadratic function**. The presence of the ax^2 term is a big hint that this is a quadratic expression. You'll also remember that the ax^2 term is a hint that **factoring** is involved for solving x.

Graphs of *quadratic functions* are called **parabolas** and have a shape that looks like an airplane wing.

Let's look at two examples.

Example 1

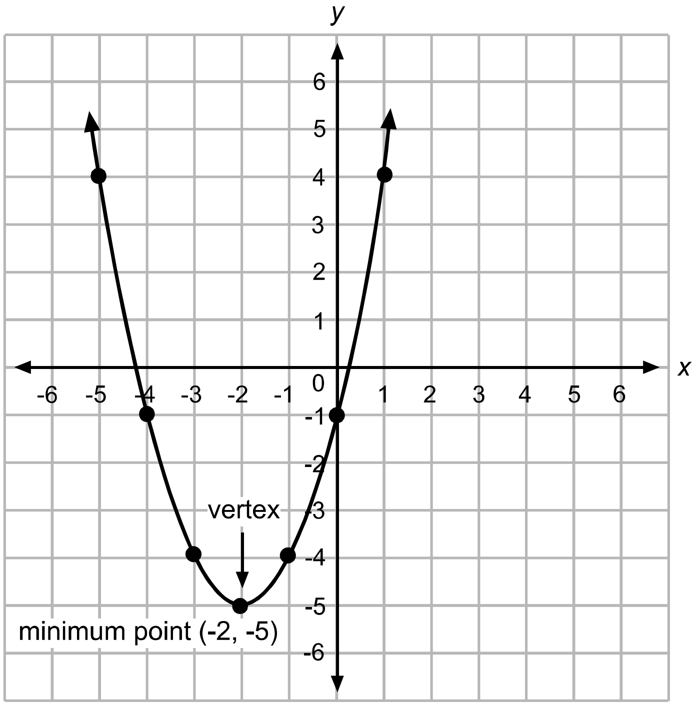
$$f(x) = x^2 + 4x - 1$$

We will use a table of values to graph this function.

Table of Values

$f(x) = x^2 + 4x - 1$	
x	$f(x)$
-5	4
-4	-1
-3	-4
-2	-5
-1	-4
0	-1
1	4

Graph of $f(x) = x^2 + 4x - 1$



We plot the points, and knowing that the graph will look like an airplane wing, we connect the dots with a smooth curve. A **coefficient** is the number part in front of an algebraic term. The *coefficient* in front of x^2 in the function $f(x) = x^2 + 4x - 1$ is understood to be a +1.

Because the x value of the coefficient is positive, the *parabola* will open upward and will have a *lowest point*, or **vertex**, called the **minimum** point.

To tell exactly where that *minimum* point will be on our graph, we use information from the equation. Remembering that $f(x) = ax^2 + bx + c$, we use $x = \frac{-b}{2a}$ to tell us the x-value of the lowest point.

So from our function $f(x) = x^2 + 4x - 1$, where $a = 1$, $b = 4$, $c = -1$, we get the following.

$$x = \frac{-b}{2a}$$
$$x = \frac{-4}{2(1)}$$
$$x = \frac{-4}{2}$$
$$\boxed{x = -2}$$

So, the minimum point occurs when $x = -2$.

Using the function again,

Remember: $f(x) = ax^2 + bx + c$
$f(x) = x^2 + 4x - 1$, where
$a = 1$, $b = 4$, $c = -1$

$$f(-2) = (-2)^2 + 4(-2) - 1$$
$$f(-2) = 4 + -8 - 1$$
$$f(-2) = -5$$

Therefore, the minimum point is (-2, -5).

Another thing we can tell from the equation $x = -2$ in the box above is the **axis of symmetry**. Recall that the graph of $x = -2$ is a vertical line through -2 on the *x-axis*. This is the line that divides the parabola exactly in half. If you fold the graph along the *axis of symmetry*, each half of the parabola will match the other side exactly.

Note that the graph is a function because it passes the *vertical line test*. Any vertical line you draw will only **intersect** the graph (parabola) at one point.

Let's look at another example.

Example 2

$f(x) = -x^2 + 2x - 3$

Notice that the coefficient of x^2 is -1.

> Because the value of the coefficient of x is negative, the parabola will open downward and have a highest point, or *vertex*, called a **maximum** point.

Find the axis of symmetry.

$x = \frac{-b}{2a}$

$x = \frac{-2}{-2}$

$\boxed{x = 1}$ ⟵ axis of symmetry

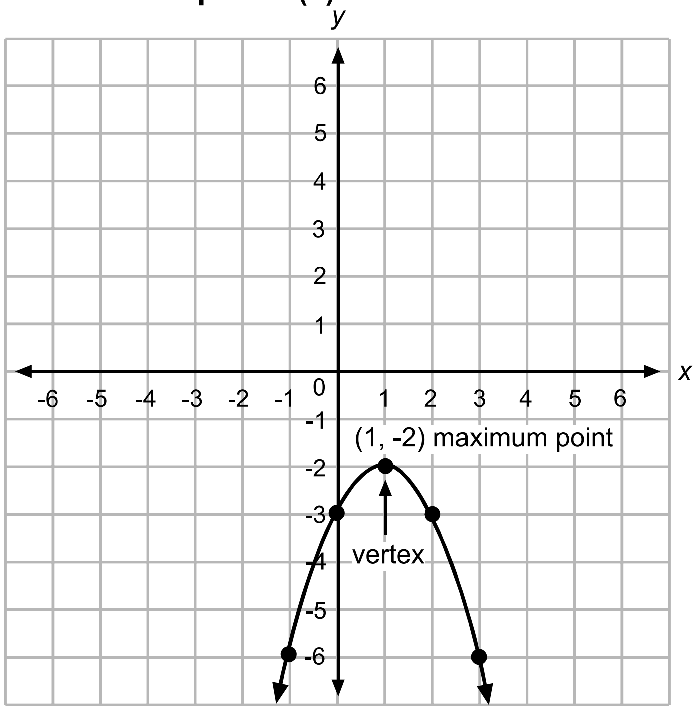
Our *maximum* point occurs when $x = 1$. Let's make a table of values (be sure to include 1 as a value for x).

Table of Values

$f(x) = -x^2 + 2x - 3$

x	$f(x)$
-1	-6
0	-3
1	-2
2	-3
3	-6

Graph of $f(x) = -x^2 + 2x - 3$



Graph the ordered pairs and connect them with a smooth curve. Note that the vertex of the parabola has a maximum point at (1, -2) and the line of symmetry is at $x = 1$.

Refer to the examples above as you try the following.

Solving Quadratic Equations

The **solutions** to **quadratic equations** are called the **roots** of the equations. In factoring *quadratic equations*, set each **factor** equal to 0 to **solve** for values of x. Those values of x are the *roots* of the equation.

Example

Solve by factoring

$x^2 + 10x + 9 = 0$ ← factor

$(x + 1)(x + 9) = 0$

Set each factor equal to 0

$x + 1 = 0$ ← **zero product property**

$x = -1$ ← add -1 to each side

$x + 9 = 0$ ← zero product property

$x = -9$ ← add -9 to each side

-1 and -9 are roots

We can also find these roots by graphing the related function $f(x) = x^2 + 10x + 9$ and finding the x-intercepts. The x-intercepts are the points where the graph crosses the x-axis, which are also known as the **zeros** of the function.

Let's see how this works.

$$f(x) = x^2 + 10x + 9$$

The equation for the axis of symmetry is as follows.

$$x = \frac{-10}{2(1)}$$

$$\boxed{x = -5}$$

$$f(-5) = (-5)^2 + 10(-5) + 9$$
$$f(-5) = -16$$

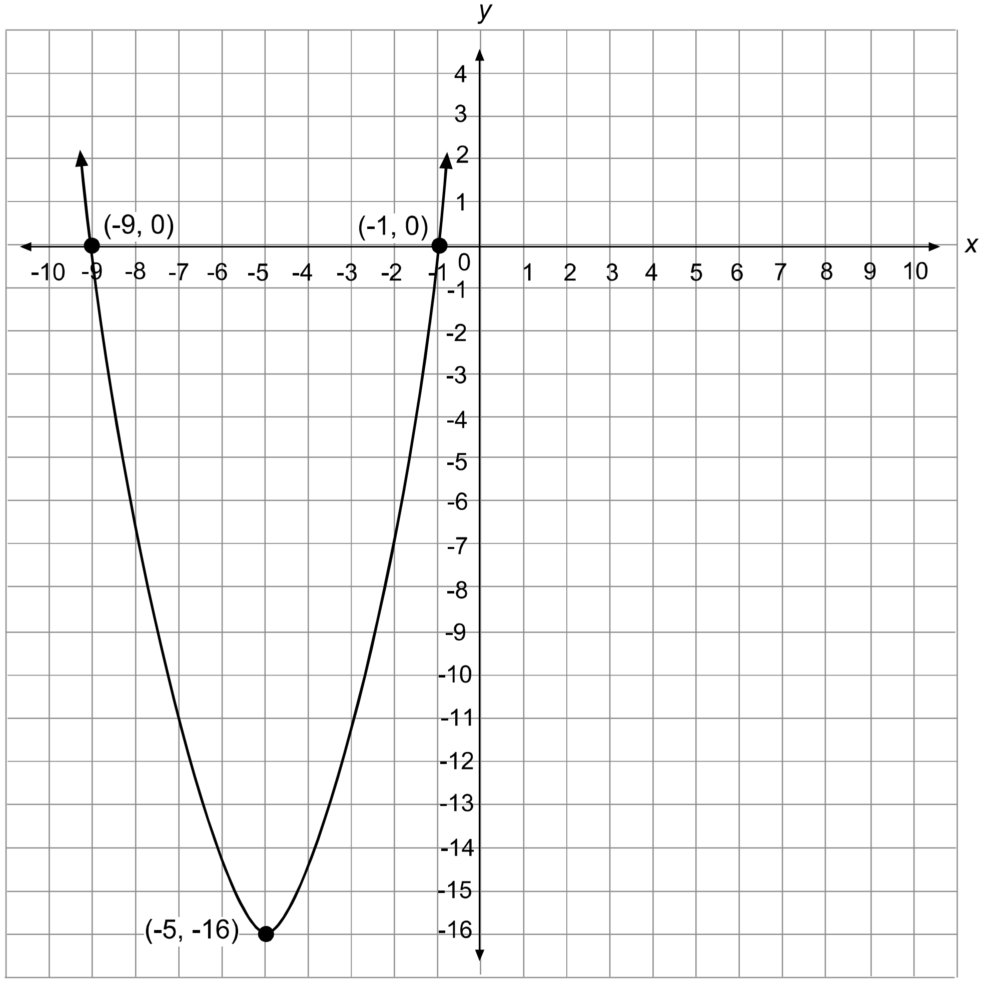
The vertex is at (-5, -16).

Find the x-intercepts by letting $f(x) = 0$.

$$0 = x^2 + 10x + 9$$

The x-intercepts are at (-9, -1). Thus the solutions are -9 and -1.

Graph of $f(x) = x^2 + 10x + 9$



You may find the solutions more efficiently by using your graphing calculator. When the x-intercepts are *not* integers, use your calculator to estimate them to the nearest integer.